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J and G_c analysis of the tearing of a highly ductile polymer

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The use of a conventional J analysis to describe the ductile tearing of thin low density polyethylene sheet is described. This is a measure of the total energy required to cause fracture. The use of the current energy release rate to determine the local dissipation rate is then given and it is shown that an initiation (plane strain) and reasonably constant propagation (plane stress) values are obtained.

ξ Input energy of system.
A Area of specimen.
B Thickness of specimen.
W Width of specimen.
a Crack length.
J J-integral.
G_c Energy release rate.
P Load.
U Energy.
C Compliance.
M Constraint factor.
σ_y Yield stress.
u Displacement.
η Dimensionless factor.

1. Introduction

Linear elastic fracture mechanics (LEFM) has been found to be useful in describing brittle fracture in polymers [1]. In these cases, the energy dissipation is very local to the crack tip so that the behaviour of the body as a whole is taken to be elastic and the energy dissipated in fracture is derived from an energy change in an elastic analysis. The practical importance of such situations is clear since they represent a lower limit of toughness for the material. Indeed, the plane strain toughness value [2] can be taken as such a bound for a given set of conditions and used in design calculations. This toughness usually includes some distortional energy and is considerably in excess of the bond scission surface work. This is because it is not possible to rupture molecular chains mechanically without first deforming them and the energy dissipated by this is inextricably linked with the work of bond scission.

Having accepted the concept of a highly constrained toughness including distortional energy, there are still cases to be considered when the toughness is much higher than this. In tough polymers, a low yield stress results in a low constraint and surface shearing can result in extra work being necessary for fracture [2, 3]. If this occurs in a contained zone around the crack, then a modified form of the LEFM analysis can give useful values for the enhanced toughness. This is not, however, the unique geometry independent lower bound value of the high constraint case and will differ for different geometries. As the extent of plastic deformation increases so the utility of LEFM declines and it is eventually unusable. Here, we must look to other approaches and similar problems in the metals field which have resulted in the J energy concept. These have been tried on polymers [4–6] but mainly when the deviation from LEFM was substantial but not gross. The correlation between modified LEFM and J is then quite good [6].

There are practical cases, and often in laboratory tests where it is necessary to evaluate grossly non-linear dissipative systems. Such situations have been treated by Andrews *et al.* [7, 8] using a generalized theory. This analysis has two quite separate aspects in that it embodies an analysis which gives J (termed $-2\,d\xi/dA$) and then seeks to compute the highly constrained lower bound

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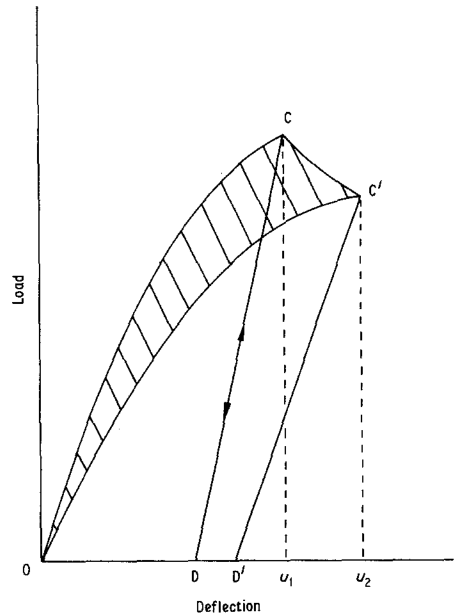

Figure 1 Load–deflection diagram for crack growth.

value by experimentally measuring the energy dissipation other than this. This approach presents very difficult and time-consuming experiments and involves some profound assumptions on the nature of the hysteresis behaviour. It is arguable that such an approach is preferable to the more direct technique of experimentally increasing the constraint of the test by, for example, using bending tests or greater thicknesses of specimen.

There remain situations, however, where the evaluation of a true fracture energy for a highly non-linear, non-elastic, system is needed, and this paper sets out to describe the analysis of such a system using quite conventional non-linear fracture mechanics. The problem to be solved is that of tearing thin, single edged notched specimens of low density polyethylene in which there is slow, stable ductile tearing.

2. The J analysis

There are several approaches to J analyses [9] but the one used here is the direct experimental method of Begley and Landes [10]. Suppose that a crack propagates in a stable manner in a body after being loaded to point C, as shown in Fig. 1, such that the displacement increases from u_1 at C to u_2 at C′ when the crack area has increased from A to $(A + \delta A)$. If the energy per unit area to propagate the crack is J, then the energy used for that purpose in the process is $J\delta A$. If $(u_2 - u_1) = \delta u$, then the external work performed is $P\delta u$, but it is not known how much of this goes into J and how much is stored or dissipated in the specimen generally.

However, at C′ the crack area is $(A + \delta A)$ and if it is assumed that the loading line for this crack length is OC′, as shown, then the shaded area between OC and OC′ represents the energy necessary to grow the crack, $J\delta A$. This is the definition of J used here and is

$$J = -\frac{\delta U}{\delta A}\bigg|_{u \text{ constant.}} \qquad (1)$$

Any external work done during crack growth goes directly into the new total energy so that J may be defined conveniently at constant displacement. This definition assumes a path independence of the crack growth in that the energy required to reach a given displacement with a given crack length is assumed not to depend on the crack growth history. This is probably a reasonable assumption for most cases, excepting those where the degree of plastic constraint varies with crack length as in some bending geometries [9]. In these cases, the effective yield stress, which varies with constraint factor, will change with crack growth and thus change J so that J will depend on how A varies.

If the body is elastic, then J represents the elastic energy release which is balanced by G_c, since equilibrium is assumed here, so that $J = G_c$. J is *not* an energy release, however, but is an energy absorption and the energy release must be computed separately. For a material exhibiting linear elasticity, the unloading line is shown as CD in Fig. 1, while at C′ there would, in general, be the line C′D′. For the purely elastic case, D and D′ coincide and the area CDC′ corresponds to the usual elastic energy release rate, G, given by

$$G = \frac{P^2}{2}\frac{\mathrm{d}C}{\mathrm{d}A} \qquad (2)$$

where C is the elastic compliance of the specimen. This change in energy is brought about by the change in the elastic compliance of the specimen due to crack growth indicated by the change of slope in the unloading lines in Fig. 1, i.e. CD to C′D′, and also the decrease in load C to C′. For an entirely elastic case, both D and D′ coincide with O and COC′ = CDC′ is G, the elastic energy

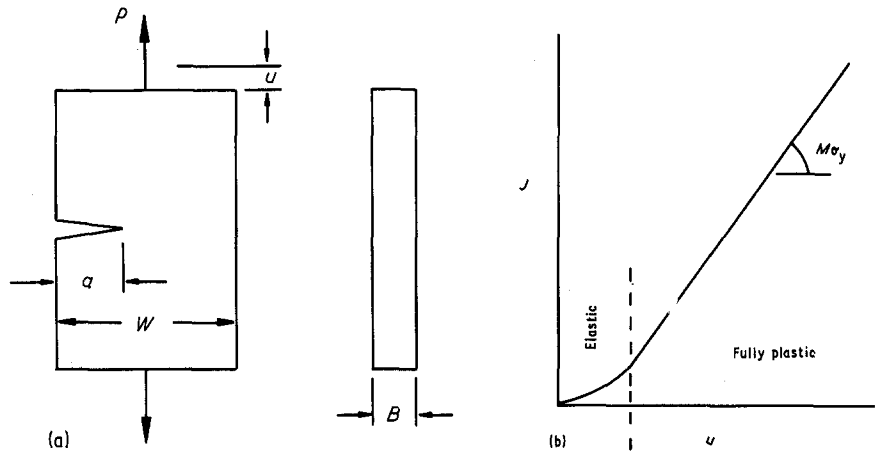

Figure 2 J for single edge notch tension.

release rate. For a fully plastic specimen, the load drop CC′ is determined and the energy release rate is given by CDD′C′. The expression for the total G may be computed from the elastic energy

$$U = \tfrac{1}{2}CP^2 \tag{3}$$

where C is the elastic compliance, u/P. G may be defined as

$$G = -\frac{dU}{dA}\bigg|_u = -\left(\frac{P^2}{2}\frac{dC}{dA} + PC\frac{dP}{dA}\right) \tag{4}$$

i.e.

$$G = -\frac{P^2}{2}\left(\frac{dC}{dA} + \frac{2C}{P}\frac{dP}{dA}\right). \tag{5}$$

For elastic unloading, we have $dP/dA = -(P/C)\,dC/dA$, giving $G = (P^2/2)\,dC/dA$, the usual result.

As an example of a J and G analysis, which is also the specimen geometry used here, it is helpful to consider the single edge notch tension case shown in Fig. 2a. The total energy for a fully yielded specimen is

$$U = B(W-a)M\sigma_y u \tag{6}$$

where M is the constraint factor. Now, $A = Ba$, so that

$$J = -\frac{\partial U}{\partial A}\bigg|_{u\ \text{constant}} = M\sigma_y u. \tag{7}$$

Thus, J should be linear in u and independent of a with a slope of $M\sigma_y$ for this fully plastic case. A linear elastic analysis shows that $J \propto u^2$ so that the J versus u curve will have the general form shown in Fig. 2b.

The total G may also be computed for this system since

$$P = B(W-a)M\sigma_y \tag{8}$$

and

$$\frac{dP}{dA} = -M\sigma_y. \tag{9}$$

From Equation 5, we have

$$G = \frac{P}{2B}\frac{dC}{da}\left[\frac{2C}{dC/da}\frac{1}{(W-a)} - 1\right] \tag{10}$$

The parameter [9]

$$\eta = \frac{(W-a)}{C}\frac{dC}{da} \tag{11}$$

is basically geometric but does depend on the nature of the deformation occurring.

It is also possible to measure the external work input during crack growth to give

$$\frac{\delta U}{\delta A} = P\frac{\delta u}{\delta A}. \tag{12}$$

3. Experiments and results

Tensile tests were performed on single edge notch specimens 50 mm x 150 mm x 3 mm extended at 1.0 cm min^{-1} in an Instron testing machine. Tests were performed with initial crack lengths varying from 5 to 40 mm and the ligament was marked in 3 mm intervals. The specimen underwent gross deformation, as shown in Fig. 3, with substantial crack blunting but a tearing of the material occurred which progressed across the specimen. As the tear passed each marker, the load–deflection record was blipped so that finally a series of load–deflection diagrams with crack length marks on them was obtained, as shown in Fig. 4. Curves could then be drawn for fixed crack length by interpolation, and these are also shown in Fig. 4.

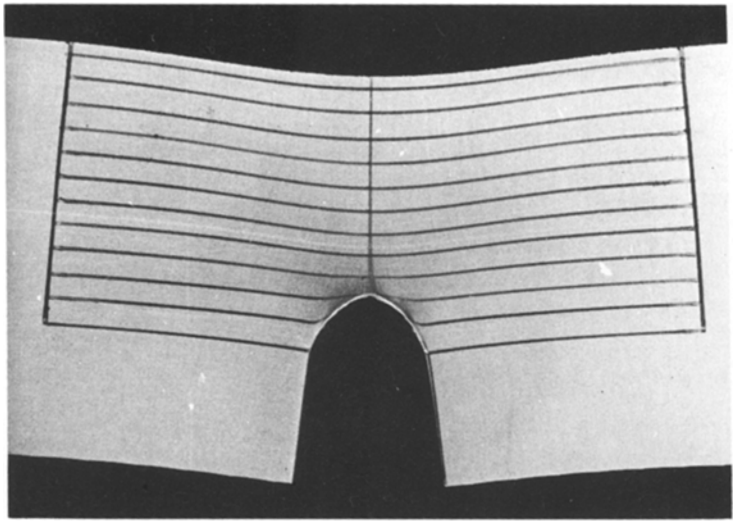

Figure 3 Single edge notch specimen tearing under load.

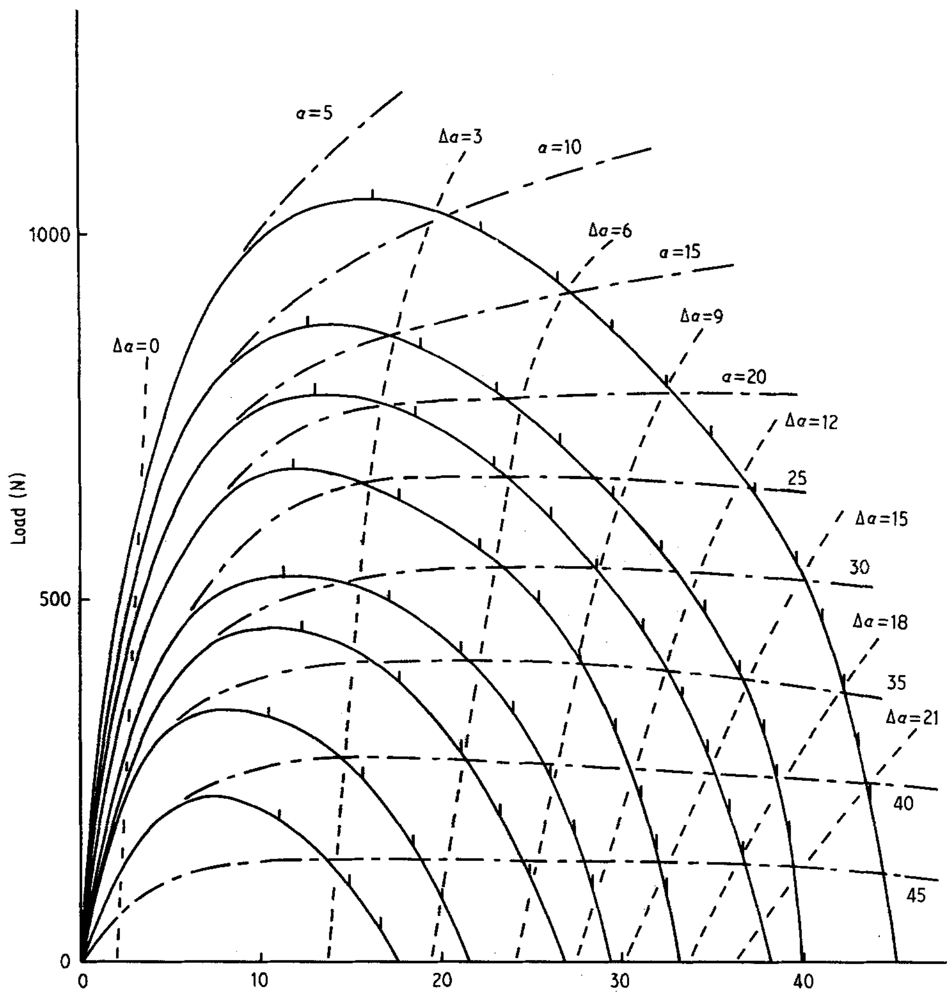

Figure 4 Load–deflection diagram. ——— experimental lines, — · — interpolated constant cracklength lines, — — — constant Δa lines.

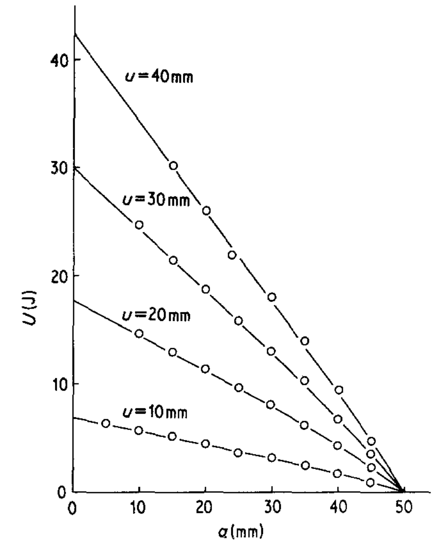

Figure 5 Energy as a function of crack length at constant deflection – J is given by the slope.

These may be graphically integrated up to different displacements and several of these lines are shown in Fig. 5. As expected for yielded specimens, they are almost straight (independent of a) except for very large a values. The resulting J versus u line is shown in Fig. 6 and this shows the expected initial elastic curvature followed by linearity. At large u values, the slope, and hence $M\sigma_y$, increases probably due to orientation hardening.

Returning to Fig. 4, it is now possible to deduce J for every point on the curves and Fig. 7 shows J versus Δa curves for the various initial crack lengths. There is rather a low initiation value which has only a slight dependence of initial crack length but there is a rapid rise of J with an increasing

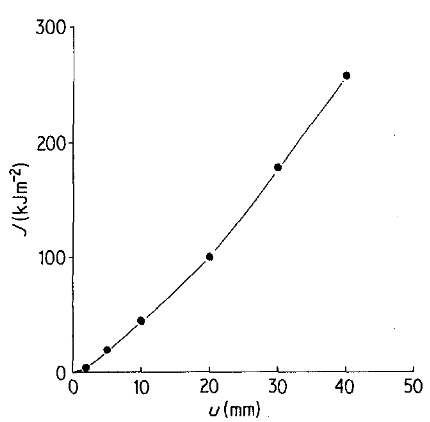

Figure 6 J as a function of displacement.

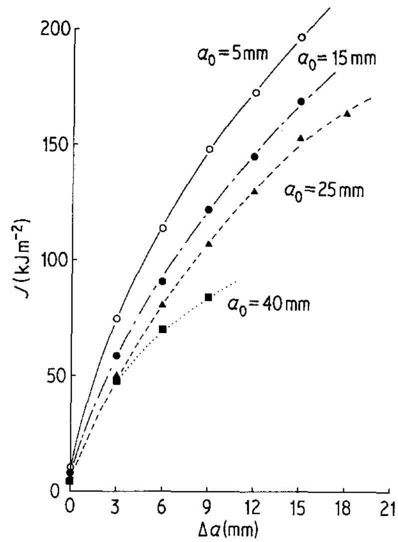

Figure 7 J as a function of crack growth for various initial crack lengths.

dependence on a. This rise is, of course, necessary for the stable tearing behaviour observed.

Fig. 8 shows the J versus Δa data for an initial crack length of 25 mm and, in addition, there is the input work rate, $\delta U/\delta A$. Initially, $\delta U/\delta A > J$ and clearly much of this work is dissipated in the specimen but in the final stages of the fracture, $\delta U/\delta A < J$. This illustrates the important point that J is the *cumulative* energy dissipation rate per unit area of crack propagation.

Much of the energy embodied in J is, in fact, dissipated *before* the crack moves and is in the body of the specimen. The local energy to actually

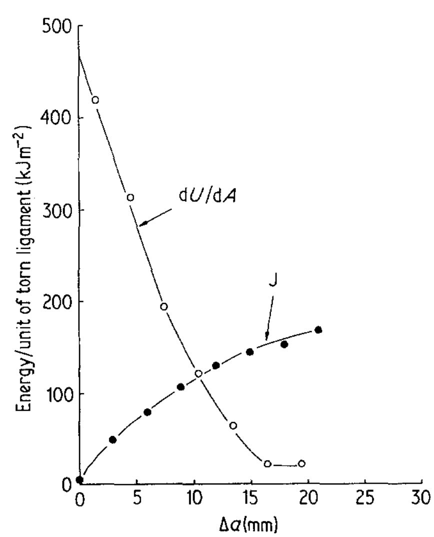

Figure 8 Energy input rate and energy dissipation as a function of crack growth (a_0 = 25 mm).

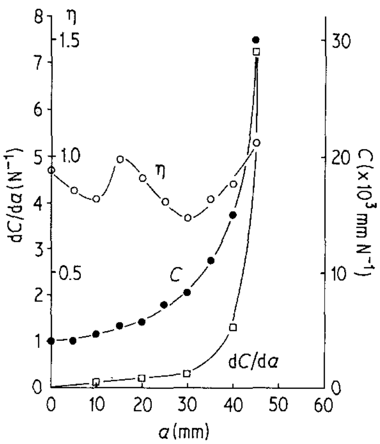

Figure 9 Initial compliance C, dC/da and η as a function of crack length.

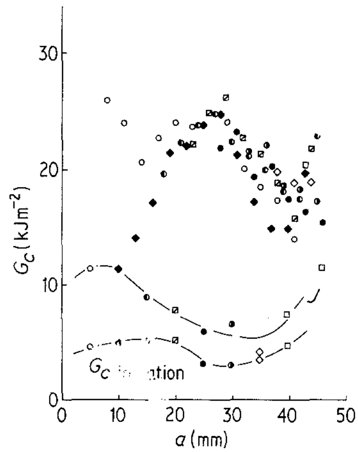

Figure 11 Energy release rates for initiation and propagation.

propagate the crack must come from the energy release rate as the crack grows. For the case of non-linear visco-elastic materials such as low density polyethylene defining the modulus to use in an energy release rate analysis is difficult because of high hysteresis and non-linearity. It is also necessary to be precise on what is meant by energy release since complete unloading does not occur locally. A sensible solution is to determine C from the initial slopes of the loading

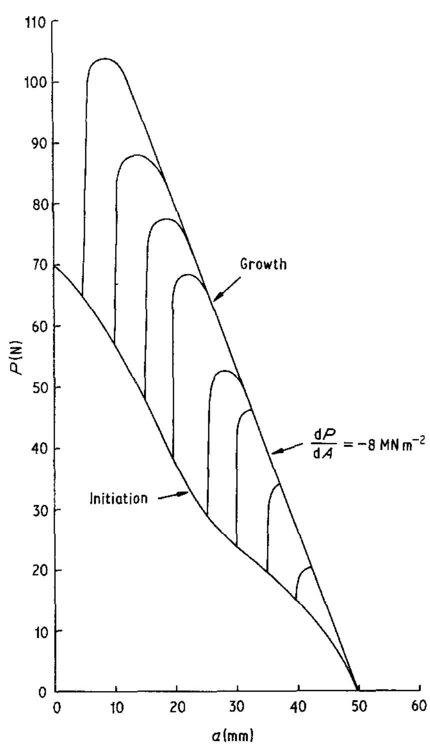

Figure 10 Load as a function of crack growth.

curves in Fig. 4 since these will embody an elastic effect appropriate to the rate of the tests and the local, current, unloading of the crack growth process. This was done and Fig. 9 shows C and dC/da derived from it, together with η. The loads from Fig. 4 are plotted versus crack length in Fig. 10 and clearly all the points lie on a constant dP/dA line required of plastic yielding other than the initiation values. The slope of the line (-8 MN m^{-2}) is, of course, the same as the J–u curve in Fig. 6 at the higher u values.

For each point on the load–deflection lines given in Fig. 4, a value of G could then be computed using the load and dC/dA for the initiation values and including η for all others. These results are given in Fig. 11, together with the initiation J values. Even at initiation, J is substantially greater than G_c, indicating considerable plastic work away from the crack tip. As the cracks propagate, G_c rises rapidly and then falls to a constant value of around 20 kJ m^{-2}. For short initial cracks, this rise is greatest, suggesting more plasticity, even local to the crack tip, for the higher loads associated with the shorter cracks. The larger initial cracks do not show this but some do rise very rapidly at the end of the specimen. This is probably due to high orientation when a small ligament is present. It is, however, clear that G_c, during propagation, varies very little particularly when the crack is well established.

4. Conclusions

These results establish that the J and G analyses work well even for this extremely ductile material. J is clearly the measure of the *total* energy to

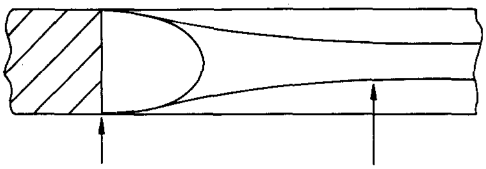

Figure 12 Tear surface profiles.

propagate the crack, while G_c is local energy dissipation to finally effect fracture. There is some dependence of G_c on crack length but there is a fairly constant initiation value of $4\,kJ\,m^{-2}$ and one for propagation of $20\,kJ\,m^{-2}$. The tear initiates with a plane front as shown in Fig. 12 and then rapidly draws in as propagation is established. The initiation G_c is thus appropriate to a plane strain condition, while that during propagation represents a fully established state of plane stress. This is supported by the fact that plane stress and plane strain toughness values determined in impact by varying specimen thickness gave similar values [11]. The plane strain values should be similar to those obtained by the Andrews method but is about a factor of ten higher than that quoted for low density polyethylene [8]. The values are likely to be sensitive to grade and this may account for the discrepancy. A direct comparison of the methods would be needed to resolve this matter.

It seems likely that the initiation (plane strain) and propagation (plane stress) G_c values are material properties in that fracture will require at least these energies to be dissipated. The J values represent the total energy necessary to affect this and are probably geometry dependent. Such values are important for design calculations where total work is needed, while for material evaluation the G_c values, together with a yield stress, may be more important. The choice of the appropriate fracture energy measure is thus determined by the goal of the analysis.

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